Automated tremor amplitude and frequency determination using a Gauss curve fitting method

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Abstract

Analysis of tremor amplitude and frequency from tremor recordings is often performed in the frequency domain from signal power spectra. Current methods for tremor amplitude estimation from power spectra allow for variation in the width of power spectrum at a specific frequency, background noise, peaks resulting from sudden voluntary body part movements or multiple peaks resulting from composed tremors. However, they do not adjust tremor amplitude when the power spectra of adjacent peaks overlap and this could lead to an overestimation of tremor amplitude. We developed a MatLab based computer application for automated analysis of tremor power spectra by a modified Gauss curve fit method. This method adjusts tremor amplitude when power spectra of adjacent peaks overlap and thus enables a more accurate estimation of tremor amplitude than previously described methods. We optimised and validated this tremor analysis application by analysing 900 tremor power spectra records of 25 patients with Parkinson’s disease.

Key words: Gauss curve, Parkinson’s disease, power spectrum, tremor
1. Introduction

Tremor is a common movement disorder, present in many diseases and occasionally in healthy individuals [1]. The main features of each tremor are frequency and amplitude. Tremor frequency is mostly dependent on the pathophysiological mechanism and is fairly stable over time; for example Parkinsonian tremor frequencies are within a relatively narrow range (3 Hz to 7 Hz) [2]. Tremor amplitude, on the other hand, shows high short and long term variability, which is also modified by the progress of disease and the effectiveness of treatment [3,4].

Current instrumental methods enable accurate tremor measurements [5-8] and assessment of tremor long term variability [4,9,10] thus justifying automated analysis of tremor frequency and amplitude. Quantitative tremor measurements can be analysed in the time domain [7,11,12] or in the frequency domain [6,13-17]. Tremor analysis in the frequency domain is often done from power spectra of tremor records. Methods for tremor amplitude estimation from power spectra have been progressively refined to allow for variation in the width of power spectrum at a specific frequency, background noise, peaks resulting from sudden voluntary body part movements or multiple peaks resulting from composed tremors [15,18-22]. However, current methods do not adjust tremor amplitude when power spectra of adjacent peaks overlap. This could lead to an overestimation of tremor amplitude.

The aim of our research was to develop a new tremor power spectra analysis method that enables a higher differentiation between adjacent power spectrum peaks representing different tremor components than previously described methods. Furthermore, this new
computer method was optimised and validated by analysing 900 tremor power spectra records of 25 patients with Parkinson’s disease (PD).

2. Computational methods and theory

A power spectrum is a plot of the portion of a signal’s power falling within a given frequency interval. There is a linear relationship between the square of tremor amplitude and the area under the corresponding peak in the tremor power spectrum [23]. When the power spectrum peak is very narrow (becomes one dimensional), there is also a linear relationship between the square of tremor amplitude and the amplitude (height) of the power spectrum. Calculation of tremor amplitude on the basis of this relationship can lead to an underestimation of tremor amplitude - the broader the power spectrum peak, the larger the error [15]. Calibration of power spectra enables the determination of mean tremor amplitude from tremor power spectra. For this purpose the area under individual peaks in tremor power spectra needs to be calculated, but peak amplitudes can also be used as an approximation.

The power spectrum of a tremor signal usually consists of one peak with high amplitude and several peaks with much lower amplitudes. The one or more peaks usually represent the one or more oscillatory components of tremor. The main tremor frequency can be estimated from the position of the highest peak [14]. Tremor amplitude can be determined from the area under the corresponding power spectrum peak [23]. Different peaks in a tremor power spectrum usually overlap at least marginally, so the area of the individual peak cannot be estimated manually (Fig. 1).
The frequency and amplitude of every peak can be determined by finding its highest point (point of highest amplitude), and then reading its position on the x- and y-axis [14]. In our program we implemented a peak-searching algorithm, which defines a peak at the point of the power spectrum, where a curve slope polarity change occurs.

The peak area estimation algorithm used in our program is based on the approximation that each curve under a peak can be represented as a Gauss curve with the highest point at the same frequency as the original peak [24].

The probability density function of a standard normal distribution (Gauss curve) can be defined by the equation:

\[ y = \frac{-x^2}{\sqrt{2\pi}} \]

In our peak fitting algorithm we used a slightly modified equation, where \( x \) represents frequency, \( m \) is the amplitude of the peak and \( a \) is the parameter which defines the width and shape of the curve:

\[ y = m \times e^{-\frac{x^2}{a}} \]

As peaks in every power spectrum vary in width and shape we could not use the same parameter \( a \) value for all peaks. We assumed that the width of each individual peak is dependent on the frequency interval between its neighbouring peaks. Therefore we calculated parameter \( a \) as:

\[ a = (x_{\text{next peak}} - x_{\text{previous peak}}) / A \]
For all peaks in a power spectrum, we used the same value for parameter $A$.

To fit a Gauss curve onto an individual peak of a power spectrum we needed to determine its amplitude, frequency and parameter $A$. The first two parameters can be calculated from the position of the function peak as already mentioned above. Parameter $A$, which influences the width of the Gauss curve, can be determined by fitting Gauss curves onto the original spectrum and calculating the difference between areas under the original and fitted curves. The area difference in this way represents the quality of curve fit.

In summary, the fitting algorithm detects the power spectrum peaks and determines the best value for parameter $A$, which causes the smallest difference between the area under the original power spectrum and the area under the generated power spectrum [24]. The algorithm for the determination of power spectrum peak amplitude, frequency and area is schematically represented on Fig. 2.

Three MatLab subroutines were used in the algorithm:

- **Peakdetect**: a custom written algorithm, which searches for peaks in the power spectrum. Input data: power spectrum (x, y). Output data: Peak (x, y). A peak is detected at each point with a curve slope polarity change.

- **INTERP1 (CUBIC) and SPLINE**: Standard MatLab functions for cubic function interpolation. As the original power spectrum and the generated Gauss curves do not have the same x values they cannot be subtracted straightforwardly. Interpolation of the original power spectrum is performed first to correct its x values.
The area of the power spectrum and generated Gauss curves is calculated as a sum of all amplitudes at all x values.

We evaluated the efficiency of the algorithm by measuring the dependence of time to curve fit on the number of data pairs in a power spectrum file (Fig. 3). The time-to-curve fit increased less than 2.5-times when the number of data pairs in a file increased 100-times from 240 to 24,000.

3. Program description and samples of program runs

Our power spectrum analysis tool (GaussPeaks) is a MatLab application, using MatLab function libraries and runs on a computer with the Microsoft Windows operating system (Fig. 4).

GaussPeaks locates all the peaks in a power spectrum, determines their frequencies and estimates with a high accuracy the amplitudes and areas of every peak. For data analysis to begin, one has to select individual files or whole directories with power spectra files (in txt format) and click on the button Start AutoFit. This starts the process of fitting Gauss curves to the original power spectrum. For each set of curves the program calculates the difference between the area of the original power spectrum and the area of the generated power spectrum. After finishing the fitting process the program saves the frequencies, amplitudes and areas of the generated power spectrum, which is the most similar to the original one, in a newly generated file.
While the calculated power spectrum (a sum of calculated Gauss curves) has a theoretically infinite number of data points, the original power spectrum has a finite number of points. To compare the two curves, the program performs an interpolation of the original curve to make the size of point positions vector equal in the original and fitted curve. The user can choose between two interpolation algorithms: spline and cubic interpolation. Alternatively, by choosing the auto-interpolation setting the program performs interpolations of the original curve using both algorithms and uses the result of the interpolation which generates a better-fitting power spectrum (Fig. 5).

The process of fitting Gauss curves into the original power spectrum consists in varying the shape of the Gauss curves by increasing the value of the curve parameter $A$ from a starting value 1 to a maximum value determined by the user (Fig. 4, Max. Iterations 200). The user can also define the size of parameter $A$ step (default size is 1, see Auto Step in Fig. 4). The size of parameter $A$ step influences the speed of the fitting process and the accuracy of the final fit (Fig. 6).

For most tremor power spectra the best fit is usually achieved with maximum parameter $A$ values (i.e. number of fitting iterations) between 10 and 50, only rarely values over 50 are needed. For this reason we implemented in the program an algorithm that terminates the fitting process after reaching the best fit (Fig. 4, Stop at Best Fit) before testing all possible parameter $A$ values. At the beginning of the fitting process, the user can set the maximum number of fitting iterations (e.g. Maximum Iterations is 200 in Fig. 4). The point of best fit is reached when the difference between the area of the original power spectrum and the area of the generated power spectrum stops decreasing and starts to increase with subsequent iterations.
In some power spectra the difference between the area of the original power spectrum and the area of the generated power spectrum decreases more than once during subsequent iterations. In such cases the smallest area difference can be calculated by performing additional iterations after best fit (e.g. Fig. 4, *Iter. After Best Fit* is 10) or calculating all possible parameter $A$ values. We tested the efficiency of the algorithm, for terminating the fitting process after reaching the best fit, on 900 tremor power spectra files and the results are summarized on Figs. 7 and 8. The higher the number of iterations the program performs after reaching a supposed best fit, the higher the probability of reaching the best possible fit (the lowest area difference between the original power spectrum and the generated power spectrum). For best fitting results, 10 to 50 more iterations should be performed after reaching a supposed best fit.

The program can search for maximum peaks in the whole power spectrum or only within a predetermined frequency interval. This option is useful when analysing tremor power spectra, where we are interested in peaks within limited frequency intervals e.g., in parkinsonian tremor only between 3-7 Hz.

To test the program we analysed 900 power spectra files of hand rest tremor recordings of 25 patients with Parkinson’s disease. Tremor measurements were performed with custom-made force transducers. Signal amplification, conversion into digital form (16 bit resolution, 100 Hz sampling rate) and recording were performed with a personal computer measuring system and a MatLab-based signal-recording program [25]. Power spectra were calculated off-line from tremor signals using the Welch algorithm and saved as text files containing two columns of data [23, 26]. The first column contained the whole scale of frequencies in the
power spectrum, the second column contained power spectrum amplitudes at discrete frequencies. An example of a tremor power spectrum analysis by GaussPeaks is shown in Fig. 9. Analysed tremor power spectra of PD patients were used to display tremor amplitude over time as shown in Fig. 10.

We evaluated the congruency between the calculated tremor amplitudes and the clinically observed tremor amplitudes that were measured by the semi quantitative TRGRS (Tremor Research Group Rating Scale) scale [27]. From tremor power spectra we calculated TRGRS scores and compared these scores to the clinically observed TRGRS scores within the same time window. There was a perfect agreement between the calculated and observed TRGRS scores (Fig. 11).

4. Discussion

When developing computer applications for tremor signal analysis, two aspects should be considered: i) mathematical methods for tremor amplitude and frequency estimation from tremor recordings and ii) the computer-human interface used for automatisation of data analysis and for data representation.

4.1 Mathematical methods for tremor analysis in time and frequency domain

Tremor often possesses features that make it difficult to analyse in the time domain, for example: long tremor signal recordings, repetitive patterns and unpredictable high amplitude noise [15]. This is why analysis in the time domain is mainly used for short tremor measurements [7,22]. Signal analysis in the frequency domain (from power spectra) has been
used as a very suitable alternative, but computerised calculations are needed and as high as possible automatisation of data analysis is preferred [15]. While the identification of tremor frequencies from a tremor power spectrum is relatively straightforward, there is no optimal method for the determination of tremor amplitude since multiple peaks can be overlapping in narrow frequency intervals and the peak’s boundaries are often not clear-cut, but gradually merge into the background.

Three methods for the determination of tremor amplitude from tremor power spectra have been described.
1. The first method used the square root of the amplitude of a power spectrum peak as an estimation of tremor amplitude [15,18-21]. This method is susceptible to the variability of the shape and width of a power spectrum peak. With power spectrum peaks of approximately the same shape and width, the area of each peak depends almost in a linear way on a peak’s height (amplitude). This linearity is lost with higher peak shape or width variability and the peak’s amplitude becomes a coarser estimate for tremor amplitude [15].

2. The second method calculates the square root of the total power spectrum area within a specified frequency interval as an estimation of tremor amplitude [22]. This approximation relies on the assumption that the tremor power spectrum peak corresponds to the majority of the total power spectrum area in a specified frequency interval. Most tremor power spectra also include other components which contribute to the total power spectrum area: background noise, peaks resulting from sudden voluntary body part movements and multiple peaks resulting from composed tremors (i.e., essential and parkinsonian tremor). The presence of these factors reduces the accuracy of tremor amplitude estimation. Some improvement can be achieved by narrowing the observed frequency interval as much as possible around the
expected tremor power spectrum peak (for example in the case of parkinsonian tremor to the interval of 3-7 Hz).

3. The third method further improves the estimation of tremor amplitude by calculating the tremor power spectrum peak area of a narrow frequency interval around the peak frequency. Two variants of this approach have been described: i) the area under a peak is calculated in a fixed frequency range between -1 Hz and +1 Hz of the peak frequency or ii) the frequency interval is defined by frequencies which have just half the amplitude of the peak [15]. This method is not influenced by voluntary body part movements (as they have a much lower frequency than tremor), but it can not discriminate the amplitudes of tremor components with closely lying peaks.

The Gauss curve fitting algorithm, used in GaussPeaks program, enables an accurate estimation of the area under each peak in the power spectrum. The accuracy of power spectrum fit was at least 95% in 93% of tested power spectra files. The accuracy of power spectrum fit decreased when the tremor peak amplitude approached the noise amplitude (in 7% of analysed power spectra files). The main advantages of GaussPeaks over the previously described methods are: i) it discriminates between adjacent peaks; ii) the area under each peak is calculated with great precision, not depending on the width or shape of each peak; iii) as the area under each peak is calculated separately there is no influence by voluntary body part movements. These features make the GaussPeaks algorithm theoretically a more accurate method for tremor amplitude evaluation than previously described methods.
4.2 Automatisation of data analysis and data presentation

Traditionally, each group working in tremor research uses its own developed software for tremor recording and analysis [28]. There are not many tremor recording and analysis applications that would appeal to users who wish to analyze tremor records in a standardized daily clinical procedure and also to users who work in the different fields of tremor research [28].

GaussPeaks is a specialized software application that offers automated, computer analysis of tremor power spectra. It can be used in combination with other tremor recording and analysis applications. GaussPeaks has new features which are especially useful when analysing great quantities of data: i) a practically unlimited number of files or directories with power spectra files can be selected and analysed in one session; ii) the file name length does not limit the total number of files that can be selected for analysis; iii) very fast analysis of power spectrum files with a high number of data pairs.

For a given power spectrum GaussPeaks calculates the frequency, amplitude and area of each individual peak within a specified frequency range. On a medium-range personal computer the program is fast enough to be used for research or clinical work. The accuracy of peak amplitude and area estimates can be adjusted by setting the maximum number of iterations, the size of parameter A step and the type of power spectrum interpolation. An optimal peak area estimation can be achieved with the preset value of at least 10 iterations after the best fit, with parameter A step size 1 and with automatic selection of the power spectrum interpolation type.
To calculate the absolute tremor amplitude value from a tremor record is not considered a trivial problem. The major reason for errors is problems with mathematical analysis of data including calibration [15]. After power spectrum calibration, GaussPeaks enables an accurate calculation of absolute tremor amplitude values (Fig. 10). With the TRGRS scale we compared tremor scores from clinical (visual) tremor evaluation (observed TRGRS scores) to tremor scores calculated from measured tremor amplitudes (calculated TRGRS) during the same time interval. There was a high degree of concordance between the tremor scores (Fig. 11). Appropriate values of measured tremor amplitudes and the concordance of measured tremor amplitudes with semi quantitative amplitude TRGRS scores confirm the validity of the tremor amplitude estimation method used in GaussPeaks.

4.3 Conclusion

We developed, optimised and validated a MatLab based computer application for automated analysis of tremor power spectra by a modified Gauss curve fit method. This method adjusts tremor amplitude when power spectra of adjacent peaks overlap and in such cases enables a more accurate estimation of tremor amplitude than previously described methods.

5. Hardware and software specifications

The GaussPeaks program runs on a personal computer with a Pentium 4 processor, 512 Mb of RAM, Microsoft Windows XP operating system and MatLab software, ver. 6.5 or higher.

6. Mode of availability of the program

The GaussPeaks program is not jet commercially available. For obtaining a testing version please contact the corresponding author.
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Figure legends

Figure 1
Schematic representation of the process of fitting Gauss curves into a tremor power spectrum.

Top graph. A representation of a tremor power spectrum with two main peaks, which overlap marginally. The area under each peak cannot be determined by a straightforward measurement.

Bottom graph. Our method for estimating peak area. A Gauss curve is fitted under each power spectrum peak. From the individual Gauss curve, the program approximates the area of the corresponding power spectrum under each peak.

Figure 2
Flow chart of the algorithm for the determination of power spectrum peak amplitude, frequency and area.

Figure 3
Dependence of time to curve fit on the number of data pairs in a power spectrum file.
**Figure 4**

Screen shot of the main window of the GaussPeaks program. *Select File(s), Select Directory, Select Directories* – enable the selection of files or directories with power spectrum data to be analysed; *Interval of interest* – frequency interval within the power spectrum, where peaks will be searched for; *Start AutoFit, Cancel Auto* – start and termination of the Gauss curve fitting process; *Stop at Best Fit* – enables/disables the feature of stopping the fitting process after the supposed best fit is found; *Iter. After Best Fit* – selects the number of iterations to be performed after the first best fit is reached; *Enable Auto-interpolation* – enables the automatic selection of power spectrum interpolation method; *Frequency, Amplitude* – show the frequency and amplitude of the current pointer position; *Divider for A* – shows the parameter A value at the point of best fit. In the presented example, the area difference between the original and calculated power spectrum (Area Diff.) is 0.16%.
Figure 5

Comparison of three power spectrum curve interpolation strategies (spline only, cubic only and best fit – spline or cubic) on the accuracy of the curve fit to the original tremor power spectrum. The accuracy of the curve fit was estimated by measuring the area difference between the original and calculated power spectrum - the lower the area difference, the higher the accuracy of the curve fit. The comparison was performed on 900 tremor power spectra files (n = 900). The graphs in the left column display an overview of the tremor power spectra area differences. The graphs in the right column give a breakdown of tremor power spectra area differences for each power spectrum curve interpolation strategy. The power spectrum area difference is lower with spline interpolation than with cubic interpolation. For example, 52% of power spectra files have an area difference <1% after spline interpolation and 42% of power spectra files have an area difference <1% after cubic interpolation. Percentage values were rounded to the nearest whole number (e.g. 11.5 % to 11 %; 11.6 % to 12 %). The optimal power spectrum curve interpolation strategy is for the program to perform interpolations of the original curves with both algorithms and to select the interpolation which generates a better-fitting power spectrum.
Figure 6
The influence of parameter A step size on the average area difference between the original and calculated tremor power spectra. The optimal size of parameter A step is 1. Values below 1 do not reduce the area difference and values above 1 increase the difference between the original and calculated tremor power spectra. The area difference represents the average area difference of 900 tremor power spectra files.

Figure 7
The influence of the number of iterations of the fitting process (n) after reaching the supposed best fit (i.e. best fit +n) on the power spectrum area difference. We performed the fitting process on the same number of data files (n = 900) but with the program set to terminate the fitting process at one, ten or fifty iterations after reaching the supposed best fit (i.e. best fit +1, best fit +10 and best fit +50). The graphs in the left column display an overview of the tremor power spectra area differences at different values of best fit +n. The graphs in the right column give a breakdown of tremor power spectra area differences for a specific best fit +n. When the program was set to terminate the fitting process at just one iteration after the best fit (best fit +1), the total number of iterations performed was quite low (less than ten) in some cases and consequently the area difference was higher than 10% in more than 10% of all cases. When the termination point was set to ten iterations after the best fit (best fit +10), the total number of iterations was higher but the number of files with the area difference higher than 10% fell from 11% to 3%. The fitting accuracy was not improved when the number of iterations after best fit was increased from 10 to 50. Percentage values were rounded to the nearest whole number (e.g. 11.5 % to 11 %; 11.6 % to 12 %).
**Figure 8**
The dependence of the difference between the area of the original power spectrum and the area of the generated power spectrum on the number of iterations performed in the fitting process. With increasing values of the number of iterations, the area difference approaches a minimum value, after which it starts to increase (Example 1). The best fit is achieved just one iteration before the area difference increases, which happens often in fewer than 20 iterations (Example 1). In such cases the algorithm generates only one solution. But as showed with the second, third and fourth example some tremor power spectra contain several local minimums in area differences (marked with an asterisk on Examples 2, 3 and 4). In such cases the algorithm can find more than one minimal-difference fit (i.e. at 13th and 73rd iteration in Example 4), depending on the preset maximal number of iterations. If the maximum preset number of iterations, or the preset number of iterations after the best fit, is too low a non optimal fit is achieved. For this reason it is recommended to continue the fitting process for at least 10 iterations after reaching the first best fit (i.e. best fit +10).
Figure 9
Tremor power spectrum analysis of a patient with Parkinson’s disease. The left half of the figure displays the input data file (file 1.txt). The first column of the input data file lists the tremor frequencies (from 1 - 24.9 Hz), the second the tremor amplitudes. After finishing the fitting process, the GaussPeaks program generates 7 output files (Output File 1 to 7). In **Output File 1** we can see the calculation results for each performed iteration: iteration consecutive number, total area of the original power spectrum (AreaOrig), total area of the generated power spectrum - sum of all Gauss curves (AreaComp), area difference between original and generated power spectrum (AreaDiff) and minimal area difference reached at the running iteration (MinDiff). We can see that the area difference is falling with consecutive iterations until iteration number 19, where the best fit is reached. With further iterations the area difference begins to rise again. **Output File 2** contains data about the fitting process, for example its duration, number of all iterations performed, minimal area difference reached, type of curve interpolation used. **Output File 3** lists all detected peaks in the power spectrum with their frequencies, amplitudes and peak area estimations. **Output Files 4 and 5** are excerpts from File 3; with File 4 showing the same data only for the peak with the highest amplitude and File 5 showing data for peaks within a predetermined frequency interval (in our example there are no peaks within frequency range from 0 to 2 Hz). **Output File 6** is a graphical representation of the original power spectrum and the best-fitting generated power spectrum. Numerical data of both power spectra are saved in **OutputFile 7**. Tremor amplitude and area values are in arbitrary units.
**Figure 10**

Time course of tremor amplitude in a patient with PD after administration of a tremor attenuating drug (apomorphine). The top graph shows the power spectrum area under the peak of tremor frequency at 3.8 Hz (peak area), the middle graph displays the angle of joint deflection in the metacarpophalangeal joint of the left index finger and the bottom graph the corresponding finger tip movement. Each data point in all three graphs represents an average value during a 30 s time window. The patient received apomorphine at t = 0 min.

**Figure 11**

Congruency between the calculated TRGRS (Tremor research group rating scale) scores from measured tremor amplitudes and clinically observed TRGRS scores for tremors during the same time interval. Each data point is an average of five measurements in 12 patients with PD. TRGRS scale: 0 = tremor not observed on visual inspection, 1 = tremor barely observed; 1.5 = tremor amplitude below 10 mm; 2 = tremor amplitude 10 - 29 mm; 2.5 = tremor amplitude 30 - 49 mm; 3 = tremor amplitude 50 - 99 mm; 3.5 = tremor amplitude 100 – 199 mm.
Fig. 1
Figure 2

ORIGINAL POWER SPECTRUM

IDENTIFICATION OF ALL PEAKS (MatLab subroutine PEAKDETECT)
- POSITION (FREQUENCY)
- HEIGHT (AMPLITUDE)

GAUSS CURVES FITTING PROCEDURE

PARAMETER A
START VALUE

GENERATION OF A GAUSS CURVE UNDER EACH PEAK
PARAMETERS:
FREQUENCY, AMPLITUDE, PARAMETER A

SUM OF ALL GAUSS CURVES = GENERATED POWER SPECTRUM

INTERPOLATION OF THE ORIGINAL POWER SPECTRUM
MatLab functions INTERP1 (CUBIC) or SPLINE

CALCULATION OF DIFFERENCE BETWEEN ORIGINAL AND GENERATED POWER SPECTRUM ESTIMATED AREA

DIFFERENCE HIGHER THAN IN PREVIOUS STEP

NO

INCREASE PARAMETER A FOR ONE STEP

YES

SAVE GENERATED POWER SPECTRUM WITH LOWEST AREA DIFFERENCE FROM ORIGINAL POWER SPECTRUM
DATA SAVED FOR EACH CONVEX PEAK:
- FREQUENCY
- AMPLITUDE
- AREA

Fig. 2
Dependance of time to curve fit on the number of data pairs in a file

Fig. 3
Fig. 4
Figure 5

Best fit - spline or cubic

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Spline - only fit

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Cubic - only fit

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Spline-only fit

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Cubic-only fit

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Fig. 6

Figure

Average area difference (%) vs. size of parameter A step.
Dependence of area difference on the number of iterations to best fit +1

Number of iterations
0 20 40 60 80 100 120
Area difference (%)
0 30 60 90 120 150

Best fit +1
(n=900)

- diff. >10% (99 files or 11%)
- diff. 5-10% (44 files or 5%)
- diff. 2-5% (115 files or 13%)
- diff. 1-2% (208 files or 23%)

Dependence of area difference on the number of iterations to best fit +10

Number of iterations
0 20 40 60 80 100 120
Area difference
0 30 60 90 120 150

Best fit +10
(n=900)

- diff. >10% (27 files or 3%)
- diff. 5-10% (36 files or 4%)
- diff. 2-5% (135 files or 15%)
- diff. 1-2% (225 files or 25%)

Dependence of area difference on the number of iterations to best fit +50

Number of iterations
0 20 40 60 80 100 120
Area difference
0 30 60 90 120 150

Best fit +50
(n=900)

- diff. >10% (27 files or 3%)
- diff. 5-10% (36 files or 4%)
- diff. 2-5% (135 files or 15%)
- diff. 1-2% (225 files or 25%)

Fig. 7
Figure 8

Dependance of area difference on the number of iterations in Example 1

Dependance of area difference on the number of iterations in Example 2

Dependance of area difference on the number of iterations in Example 3

Dependance of area difference on the number of iterations in Example 4

Figure 8
INPUT DATA

file 1.txt

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<td>76,790</td>
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<td>2,140</td>
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<td>76,790</td>
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<td>0,297</td>
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</table>

OUTPUT DATA

Output File 1

<table>
<thead>
<tr>
<th>N of iterations</th>
<th>time</th>
<th>autostep</th>
<th>area diff</th>
<th>interpol</th>
</tr>
</thead>
<tbody>
<tr>
<td>File 1.txt</td>
<td>29</td>
<td>4,47 s</td>
<td>*0,3870%</td>
<td>Cubic</td>
</tr>
</tbody>
</table>

Output File 2

<table>
<thead>
<tr>
<th>File name</th>
<th>Frequency</th>
<th>Amplitude</th>
<th>Gauss area</th>
</tr>
</thead>
<tbody>
<tr>
<td>File 1.txt</td>
<td>3,7000</td>
<td>0,6473</td>
<td>7,2566</td>
</tr>
<tr>
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<td>8,0000</td>
<td>0,0289</td>
<td>0,3258</td>
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<td>File 1.txt</td>
<td>15,7000</td>
<td>0,0007</td>
<td>0,0074</td>
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<td>0,0002</td>
<td>0,0019</td>
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<td>File 1.txt</td>
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<td>File 1.txt</td>
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</tbody>
</table>

Output File 3 (a list of all peaks with their corresponding freq., ampl, and Gauss area)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,700000</td>
<td>0,647337</td>
<td>7,256636</td>
</tr>
</tbody>
</table>

Output File 4 (frequency, amplitude and Gauss area of the largest peak)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,700000</td>
<td>0,647337</td>
<td>7,256636</td>
</tr>
</tbody>
</table>

Output File 5 (number of peaks in frequency range 0-2 Hz)

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Amplitude</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>none</td>
<td>none</td>
</tr>
</tbody>
</table>

Output File 6 (Original and Gauss fit image) and File 7 (ASCII data of Original and Gauss fit image)
Fig. 10